LAMINAR FREE CONVECTION ON A VERTICAL NONISOTHERMAL PLATE WITH VIGOROUS BLOWING

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Asymptotic solutions are obtained for the equations of a dynamically similar boundary layer in the case of natural convection on a vertical nonisothermal plate with vigorous blowing.

Asymptotic solutions were obtained in [1, 2] for the equations of motion to describe forced convection in intensive blowing over an isothermal surface. It was established in these works that an inviscid vortical boundary layer with a constant temperature T_w is formed close to the surface. Velocity and temperature vary up to their values in the environment within a relatively narrow, viscous region adjacent to an inviscid layer. A boundary flow of similar structure was seen on a vertical plate with vigorous blowing in [3]. Presented below are asymptotic solutions of equations of a dynamically similar boundary layer with an exponential distribution of wall temperature $T_w - T_\infty \sim x^n$ $(n \geqslant 0)$.

With a change in injection rate according to the law $V_w \sim x^{(n-1)/4}$, the system of boundary-layer equations describing free convection on a vertical plate reduces to the ordinary differential equations [4]:

$$F''' + (n+3)FF'' - (2n+2)F'^2 + \theta = 0, \quad (1)$$

$$\theta'' + \Pr[(n+3)F\theta' - 4nF'\theta] = 0 \quad (2)$$

with the boundary conditions

$$\eta = 0 \;\; F = -\alpha,\; F' = 0,\; \theta = 1; \quad \eta = \infty \;\; F' = \theta = 0. \quad (3)$$

To construct the solution in the boundary region of the flow, we change over to the variables $\xi = \eta/\alpha$ and $f = F/\alpha$ in (1)-(3):

$$\frac{1}{\alpha^2} f''' + (n+3)ff'' - (2n+2)f'^2 + \theta = 0, \quad (4)$$

$$\frac{1}{\alpha^2} \theta'' + \Pr[(n+3)f\theta' - 4nf'\theta] = 0, \quad (5)$$

$$\xi = 0 \;\; f = -1,\; f' = 0,\; \theta = 1;\; \xi = \infty \;\; f' = \theta = 0. \quad (6)$$

In the case of high values of the injection parameter α, the solution of the problem (4)-(6) can be represented in the form of the series

$$f = \sum_{k=0}^{\infty} \frac{f_k(\xi)}{\alpha^{2k}}, \quad \theta = \sum_{k=0}^{\infty} \frac{\theta_k(\xi)}{\alpha^{2k}}, \quad (7)$$

where the functions f_k and θ_k are found from the chain of equations

$$k = 0 \;\; (n+3) f_0 f_0'' - (2n+2) f_0'^2 + \theta_0 = 0, \quad (8)$$

$$(n+3) f_0 \theta_0' - 4n f_0' \theta_0 = 0, \quad (9)$$

$$k \geqslant 1 \;\; (n+3)\left(\sum_{i=0}^{k} f_{k-i} f_i''\right) - (2n+2)\left(\sum_{i=0}^{k} f_i' f_{k-i}'\right) + \theta_k = -f_{k-1}''', \quad (10)$$

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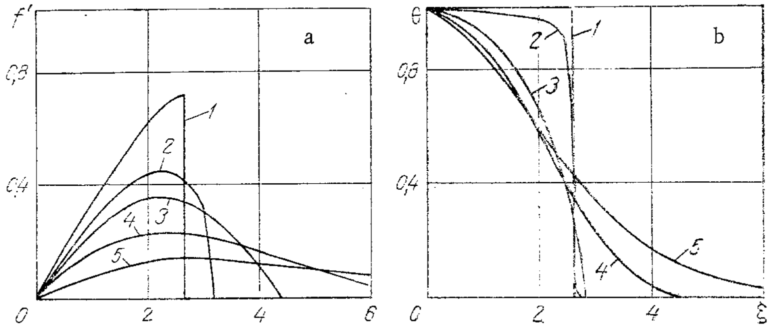

Fig. 1. Distribution of velocity f' and temperature θ: a) 1-5) n = 0, 0.5, 1.0, 3.0, 10.0; b) 1-5) n = 0, 0.008, 0.125, 1.0, 10.0

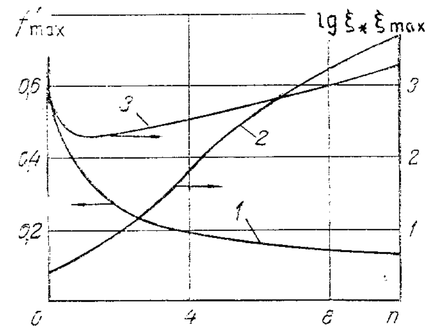

Fig. 2. Effect of parameter n on f'_{max} (1), ξ_* (2), and ξ_{max} (3).

$$\mathrm{Pr}\left[(n+3)\left(\sum_{i=0}^{k} f_i\theta'_{k-i}\right) - 4n\left(\sum_{i=0}^{k} f'_i\theta_{k-i}\right)\right] = -\theta''_{k-1} \tag{11}$$

with the boundary conditions

$$\xi = 0 \quad f_0 = -1,\ f'_0 = 0,\ \theta_0 = 1; \tag{12}$$

$$\xi = 0 \quad f_k = f'_k = \theta_k = 0. \tag{13}$$

The order of Eqs. (8)-(11) is one less than the order of the initial equations. Thus, the values of $f''_k(0)$ and $\theta'_k(0)$ for any $k \geqslant 0$ and, hence, friction and heat flux on the wall, similar to the problem of forced convection with vigorous injection [5, 6], can be found by solving system (8)-(11) with boundary conditions (12) and (13) without allowing for the conditions on the outer boundary of the boundary layer:

$$F''(0) = \frac{1}{\alpha(n+3)} + \frac{1}{\alpha^5}\left[\frac{1}{(n+3)^4} - \frac{4(2n+1)}{(n+3)^5} - \frac{4n}{\mathrm{Pr}(n+3)^5}\right] + O\left(\frac{1}{\alpha^9}\right),$$

$$\theta'(0) = -\frac{4n}{\alpha^3\,\mathrm{Pr}(n+3)} + O\left(\frac{1}{\alpha^7}\right).$$

The integral of Eq. (9) has the form

$$\theta_0 = |f|^{\frac{4n}{n+3}}. \tag{14}$$

Then calculation of the velocity and temperature distributions in the zeroth approximation with respect to the small parameter $1/\alpha^2$ reduces to the solution of Eq. (8) with allowance for (14). Figure 1 shows graphs of the functions $f_0(\xi)$ and $\theta_0(\xi)$. The thickness of the inviscid layer ξ_* is determined from the condition $f(\xi_*) = 0$. It can be seen from Fig. 1 that an increase in the parameter n is accompanied by erosion of the boundary, which is distinctly present with an isothermal wall and which separates the injected moving gas with the temperature T_w from the quiescent medium with the temperature T_∞. At n = 1, the distribution of longitudinal velocity is symmetrical relative to the maximum and has the form

$$f' = \frac{\sqrt{2}}{4}\sin\left(\frac{\sqrt{2}}{2}\xi\right).$$

Figure 2 shows the dependence of the maximum value of the velocity profile f'_{max}, the thickness of the inviscid layer ξ_*, and the coordinate of the maximum velocity ξ_{max} on the parameter n. The velocity maximum decreases with an increase in n, while the thickness of the boundary layer increases. The minimum of ξ_{max} corresponds to n = 1.

At n = 0, in contrast to n > 0, heat flux to the wall cannot be determined from the expansion (7), since $\theta'(0) = O(1/\alpha^{2k})$ at all k. Instead, the heat flux should be found directly from the solution of Eq. (2)

$$\theta = 1 - \int_0^\eta \exp\left(-3\Pr\int_0^{\eta_1} F d\eta_2\right) d\eta_1 \Big/ \int_0^\infty \exp\left(-3\Pr\int_0^{\eta_1} F d\eta_2\right) d\eta_1,$$

from which

$$\theta'(0) = -1\Big/\int_0^\infty \exp\left(-3\Pr\int_0^{\eta_1} F d\eta_2\right) d\eta_1. \tag{15}$$

In the case of vigorous blowing, the asymptotic value of the integral in Eq. (15) can be determined

$$\int_0^\infty \exp\left(-3\Pr\int_0^{\eta_1} F d\eta_2\right) d\eta_1 = \alpha\left[\int_0^{\xi_*} \exp\left(-3\Pr\alpha^2\int_0^{\xi_1} f d\xi_2\right) d\xi_1 + \right.$$
$$\left. + \int_{\xi_*}^\infty \exp\left(-3\Pr\alpha^2\int_0^{\xi_1} f d\xi_2\right) d\xi_1\right] = \exp\left(-3\alpha^2\int_0^{\xi_*} f d\xi\right) \times$$
$$\times\left(\sqrt{\frac{2^{1/2}\pi}{6\Pr}} + \frac{1}{3\Pr f_\infty}\right). \tag{16}$$

From Eq. (8) we have

$$\int_0^{\xi_*} f d\xi = \sqrt{2}\int_{-1}^{0} \frac{f df}{\sqrt{1-f^{4/3}}} = -\frac{3}{2\sqrt{2}} B\left(\frac{2}{3}; \frac{1}{2}\right), \tag{17}$$

where $B(x; y) = \int_0^1 t^{x-1}(1-t)^{y-1} dt$ is the beta function.

The value of $f_\infty = F(\eta \to \infty)$ is determined from the problem of asymptotic growth of the inviscid boundary solution with the boundary conditions away from the plate examined in [1]:

$$F''' + 3F''F - 2F'^2 + \theta = 0,$$
$$\theta' + 3\Pr F\theta' = 0, \tag{18}$$
$$\eta_1 \to -\infty \; F = 0, \; F' = 1/\sqrt{2}, \; \theta = 1; \; \eta_1 \to \infty \; F' = \theta = 0,$$

where $\eta_1 = \eta - \eta_*$.

Allowing for Eqs. (16) and (17), the expression for heat flux on an isothermal vertical plate with vigorous injection takes the form

$$\theta'(0) = -\left(\sqrt{\frac{2^{1/2}\pi}{6\Pr}} + \frac{\alpha}{3\Pr F_\infty}\right)^{-1} \exp\left(-\frac{9}{2\sqrt{2}}\alpha^2 B\left(\frac{2}{3}; \frac{1}{2}\right)\right). \tag{19}$$

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DISCHARGE OF A TURBULENT JET IN A MONODISPERSE FLUIDIZED BED

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The velocity of initial circulation is calculated and explanations are given for the mechanism of jet collapse and the existence of bubble and jet regimes.

Apparatus with fluidized beds of granular materials are widely used in industry, thanks to the high rate at which transport processes take place within them. In several processes, such as the granulation of solutions and melts in fluidized-bed apparatus, a turbulent jet inside the fluidized bed is used. A great many experimental studies have now been done on the development of the jet. These investigations are adequately detailed in [1]. Among the numerous effects seen in the interaction of the jet with the fluidized bed, the authors focused on the following: A dome-shaped gas jet is formed when a stream is discharged into a fluidized bed; when the ratio of the height of the jet to the height of the bed $x_f/H < 0.6$, the discharge takes place in the so-called "bubble" regime, accompanied by periodic collapse of the jet with the formation of bubbles; when the ratio $x_f/H > 0.6$, discharge occurs in the "jet" regime, with the boundaries of the jet being stationary and collapse of the jet occurring with greater frequency at the very edge of the nozzle. This can be seen quite clearly with the aid of high-speed photography.

We attempted to explain the physical mechanism of collapse of the gas jet and obtain quantitative relations establishing the conditions of this phenomenon.

The discharge of a stream into a bed of granular material was studied theoretically in [1, 2] and other works, and reliable results were obtained for the case of stationary beds. However, no explanation was found for the formation of the gas bubbles. It was only shown in [2] that "excessive constriction" of the stream, with the formation of bubbles, occurs at that area of the jet where the radial velocity component of the gas changes sign.

Below we present a method of analyzing the above phenomenon on the basis of a qualitative theory of differential equations describing the motion of particles of a granular material in a turbulent jet.

We will examine a circular, vertical, axisymmetric stream injected into a fluidized bed of particles of a narrow size fraction. For the sake of specificity in subsequent discussions, we will consider the boundaries of the gas jet (or the boundaries of the zone of discharge of the stream) to be the surface formed by the closest points to the stream axis at which the velocity of the gas is equal to the free-fall velocity of the particles at the corresponding porosity. We will also assume that the porosity and, thus the free-fall velocity

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